Telecom Network Systems

Wavelength Dispersion

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One of the problems of long distance communication is so-called dispersion, the property that the speed of propagation – 'phase velocity' $v = c/n$ – depends on the frequency or wavelength of the light. Related, 'chirp' refers to the property that an increasing (or decreasing) frequency is received over time:

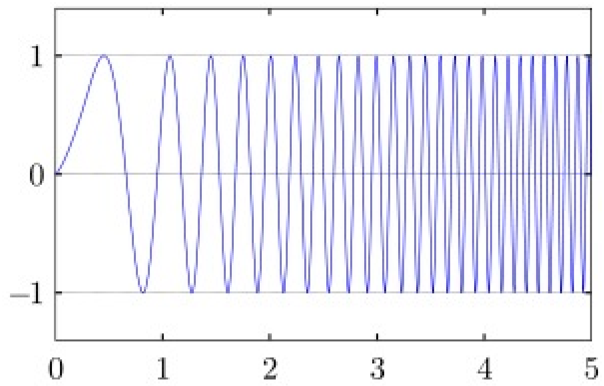

Fig. 1. Chirp

Translated to our telecommunications, if at the same time at the source a signal containing all frequencies is transmitted (like in a square wave), and the propagation speed of these waves depends on the frequency, a distorted, non-square wave, signal will be received on the other hand. Up to the point that the distortion introduces errors into our signal, i.e., misinterpretation of bits.

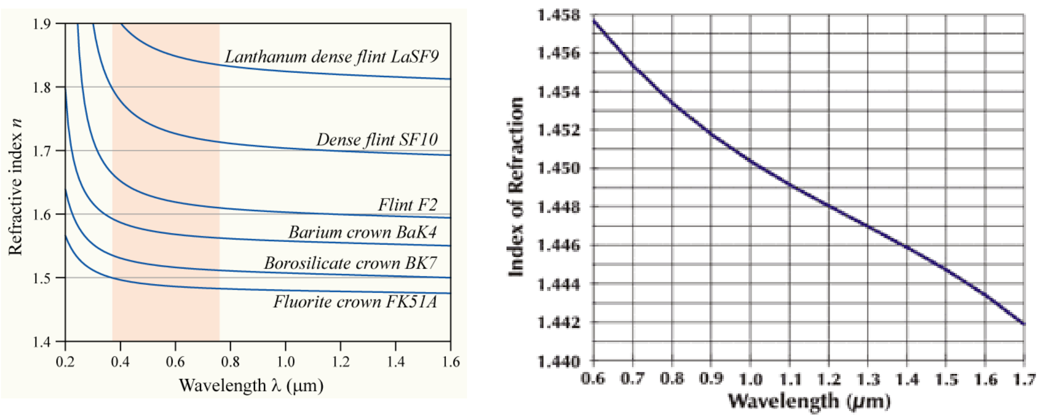

Fig 2. Dispersion in glasses, dependence of refractive index on wavelength
(Wikipedia[dispersion], http://myintro91.blogspot.pt/2012/04/optical-fiber-dispersion.html)

More important for our telecommunications is the 'group velocity' of the signal which is the speed of the power (or 'envelope' of the waves). (Wikipedia[dispersion]: The group velocity v_g is often thought of as the velocity at which energy or *information* is conveyed along the wave).

$$v_g(\lambda)=\frac{c}{\left(n-\lambda\frac{dn(\lambda)}{d\lambda}\right)}=\frac{c}{n(\lambda)}\frac{1}{\left(1-\frac{\lambda}{n}\frac{dn(\lambda)}{d\lambda}\right)}=\frac{c}{n_0}\quad , \tag{1}$$

with n_0 the extrapolated refractive index at $\lambda = 0$.

Exercise 1) Determine the phase and group velocities in glass at 1.5 μm (Fig. 2b).

Note that the group velocity itself can also depend on wavelength. There is therefore a group-velocity dispersion, defined as the second derivative of the refraction index:

$$D=-\frac{\lambda}{c}\frac{d^2n(\lambda)}{d\lambda^2}\quad . \tag{2}$$

(Unit: s/m^2). This causes a pulse to spread over time, similar to diffusion.

Fig 3. Spreading of a Gaussian pulse.

Exercise 2) Determine the parameter D in glass at 1.5 μm.

A Gaussian pulse that starts with a width T_0 has at a distance z a width $T(z)$ given by

$$\frac{T(z)}{T_0}=\sqrt{1+\left(\frac{z}{L_D}\right)^2}\quad , \tag{3}$$

with the diffusion length L_D given by

$$L_D=\frac{T_0^2}{\beta_2}\quad , \tag{4}$$

with

$$\beta_2\equiv\frac{d^2\beta}{d\omega^2}=-\frac{\lambda^2}{2\pi c}D\quad , \tag{5}$$

(Unit: s^2/m), with

$$\beta(\omega) \equiv \frac{\omega}{v(\omega)} = \frac{n(\omega)\omega}{c} \quad . \tag{6}$$

(http://www.cpeu.dk/data/uploads/teaching/34130_e2009_cpeu_dispersion_sim.pdf)

Exercise 3) Prove the above equation 5.

Exercise 4) What is the diffusion length for a T_0 = 7.5 ps pulse in the glass of Fig. 2b?

Exercise 5) What length can a cable have for such pulses spread 20 ps apart before the pulses start overlapping and become indistinguishable?

There is also often mentioned the concept of 'chromatic dispersion', which describes an effect in the frequency domain:

Adapted from http://www.fiberopticsonline.com/doc/understanding-and-measuring-chromatic-dispers-0002:

> **Chromatic dispersion** is the combined results of two different effects: **material dispersion** and **waveguide dispersion**. In silica glass, the speed of the light (refractive index, *n*) is dependent on the wavelength of the signal. **Material dispersion describes the spreading of an optic pulse due to the different speeds of the optical frequencies that make up that pulse**.
>
> **Waveguide dispersion refers to differences in the signal speed depending on the distribution of the optical power over the core and cladding of the optic fiber**. As the frequency of the optical signal decreases, more of the optical signal is carried in the cladding, which has a different refractive index than the fiber core. So, as in material dispersion, different frequencies within an optical pulse travel at different speeds. Material dispersion and waveguide dispersion tend to have opposite effects (see the figure below). Fiber manufacturers can manipulate these effects to change the location and slope of the chromatic dispersion curve.

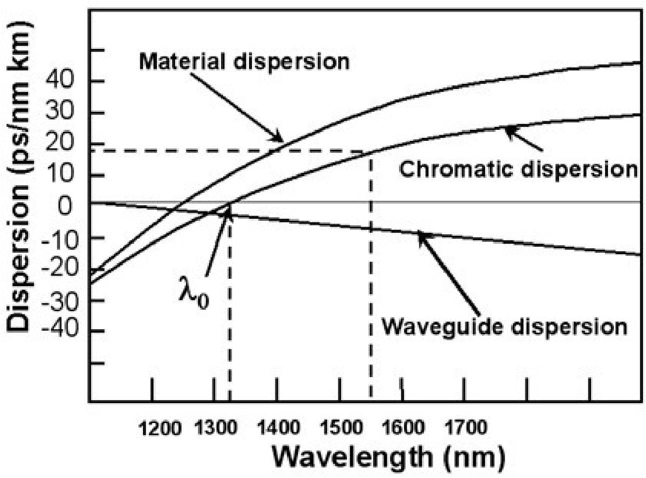

Figure 4. Chromatic dispersion is the combined result of material dispersion and waveguide dispersion, which tend to have opposite effects.

The unit of measurement for chromatic dispersion is ps/(nm km), which indicates that a pulse with a spectral width of one nanometer will spread out by one picosecond for every kilometer it travels. For example, to calculate the dispersion of a 1550-nm pulse with a 20-pm (0.02 nm) spectral width (FWHM) as it travels down a 10-km length of fiber that has a dispersion of 17 ps/nm km at 1550 nm, we would calculate:

$$(17 \text{ ps/nm km}) \times (0.02 \text{ nm}) \times (10 \text{ km}) = 3.4 \text{ ps}$$

When single-mode fibers first became commercially available, a single 1310 nm signal was transmitted, so 1310 nm was targeted to be the zero dispersion wavelength (λ_0). At this wavelength chromatic dispersion is minimized. In these fibers, known as "standard" fibers, dispersion gradually increases above 1310 until it reaches approximately 17 ps/(nm km) at 1550 nm.

Exercise 6) Calculate the chromatic dispersion (assuming no waveguide dispersion) for a 20-km cable made of the glass of Fig 2b, for a pulse of 1.5 μm and 20 pm spectral width.

Exercise 7) Plot the pulse shape of a 7.5 ps Gaussian-envelope pulse (with a carrier at 1.5 μm) after 10 km.